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GROUP THEORY.

8. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

In a chess tournament between eight players, there are seven rounds, the eight players being paired in each round, each pair to be matched once and but once in the tournament. List the possible programs, different except as to notation, *i. e.*, not transformable into each other by a substitution on eight letters. Give the number of conjugate programs of each representative retained.

Note by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

All possible solutions may be transformed into one of the six types given below. For type *F* thanks are given to Dr. Dickson. He kindly sent two solutions for comparison, by which was detected the omission of this type from the final tabulation. The results given have since been carefully checked.

<i>A</i>	15	26	37	48	<i>B</i>	15	26	37	48	<i>C</i>	15	26	37	48
	16	25	34	78		16	23	45	78		16	23	45	78
	17	24	35	68		17	28	35	46		17	28	35	46
	18	23	45	67		18	25	36	47		18	25	36	47
	12	38	47	56		12	34	58	67		12	34	57	68
	13	28	46	57		13	24	57	68		13	24	58	67
	14	27	36	58		14	27	38	56		14	27	38	56
<i>D</i>	15	26	37	48	<i>E</i>	15	26	37	48	<i>F</i>	12	38	47	56
	16	23	45	78		16	23	45	78		13	24	58	67
	17	28	35	46		17	24	35	68		14	35	26	78
	18	25	34	67		18	25	34	67		15	46	37	28
	12	36	47	58		12	36	47	58		16	57	48	23
	13	24	57	68		13	28	46	57		17	68	25	34
	14	27	38	56		14	27	38	56		18	27	36	45

Remarks by the PROPOSER.

Dr. Safford has reduced the possible types to *A—F*, but has not proved that no two of these are equivalent. I shall prove that this is true, and at the same time answer the remaining questions proposed in the problem.

A very obvious remark shows that *A* is equivalent to no one of the other types. Set $a_1 = (15)(26)(37)(48)$, $a_2 = (16)(25)(34)(78)$, ... Then $a_1, \dots, a_7$ together with identity form a commutative group of order 8 generated by a_1, a_2, a_3 , with $a_4 = a_2 a_3$, $a_5 = a_1 a_2$, $a_6 = a_1 a_3$, $a_7 = a_1 a_2 a_3$. But the product of the substitutions corresponding to the first and second lines in *B* (or *C, D, E*) is $(1472)(3856)$, so that we reach no group; similarly for *F*.

We proceed to main problem, that of finding the group of all substitutions*

*For types *A, C*, and *D*, Dr. Safford found certain of the substitutions in the course of his reductions, and kindly placed them at my disposal.

leaving a given type unaltered. The order of the group is not the same for any two types, so that no two are equivalent.

For type *A*, the group is triply transitive, of order 1344, generated by $S=(257)(346)$, $T=(36)(58)$, $V=(1537)(46)$. Obvious products of these replace 1 by 1, ..., or 8. Now S and V transform T into $R=(34)(78)$ and $P=(47)(38)$. Obvious products of these five leave 1 fixed and replace 2 by 2, ..., 8; others leave 2 and 3 fixed and replace 3 by 3, ..., 8. Now $R^{-1}TR=(46)(57)$, which is transformed into $(45)(67)$ by P . Hence there exist substitutions leaving 1, 2, 3 fixed and replacing 4 by 4, 5, 6, or 7, but not by 8, since 8 is necessarily fixed by the fifth line of *A*. When 1, 2, 3, 4 are fixed, all are fixed. Hence the order is 8.7.6.4.

For type *C*, the group is simply transitive, of order 24, generated by $R_1=(126)(385)$, $S_1=(267)(345)$, $T_1=(13)(24)(57)(68)$. There exist substitutions, leaving *C* unaltered, of the form $(1)(2x...)$ for $x=2, 6, 7$, but not for $x=3, 4, 5, 8$. When this is verified for $x=3$ and 8, it follows for $x=4, 5$. Thus, if $W=(1)(24...)$ occurred, then would $WS_1^{-1}=(1)(23...)$ occur. Since identity alone leaves 1 and 2 fixed, the order is 8.3.

For type *D*, the group is simply transitive, of order 96, generated by $(1254)(3678)$, $(13)(57)$, $(24)(68)$, $(537)(648)$, the first transforming the second into $(26)(48)$. Hence there occur substitutions $(1)(2x...)$ for $x=2, 4, 6, 8$. I find that no substitution $(1)(23...)$ leaves *D* unaltered. Hence this is true for $(1)(25...)$ and $(1)(27...)$, in view of the last generator. If, for y even, $(1)(2)(3y...)$ occurred, then by transforming by $(24)(68)$, $(26)(48)$ or by their product, we would reach $(1)(32...)$, whereas its inverse does not occur. When 1, 2, and 3 are fixed, all are; hence the order is 8.4.3.

For the type *E*, the group is transitive, of order 64, and is generated by $(1472)(3856)$, $(1876)(2345)$, $(17)(68)$, $(15)(37)$, $(17)(35)$. Combinations of the first two replace 4 by 1, ..., 8. A substitution which leaves 4 fixed must replace 6 by 6 or 8. Now $(4)(6)(5x...)$ occurs if and only if $x=1, 3, 5, 7$. If 1, 4, 6 are fixed, all are. The order is 8.2.4.

For type *F*, the group is intransitive, of order 42, and generated by (2345678) , $(346)(587)$, $(38)(47)(56)$, being the metacyclic group commutative with the cyclic G_7 .

For type *B* it was found that $(28)(46)$ is the only non-identical substitution not altering *B* and leaving 1 fixed. Another substitution occurring is $(15)(37)$. Hence the group is of order $4n$, $n=1, 2, 3$, or 4. In any event, the order is less than the orders in the previous cases.

The number of conjugate programmes of type *A* is $8! \div 1344 = 15$.

MISCELLANEOUS.

158. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

An ingot of pure gold was melted at the Mint and then 10 ounces were taken out and 10 ounces of pure silver added and the contents of the melting pot